Onda cinematica

Equazioni del moto e di continuità:

$$\begin{cases} \frac{\partial H}{\partial x} = -\frac{1}{g}\frac{\partial U}{\partial t} - j \\ \frac{\partial \Omega}{\partial t} + \frac{\partial Q}{\partial x} = 0 \end{cases}$$

Introduciamo la scala di deflusso $Q = k\Omega^m$:

$$\frac{\partial \Omega}{\partial t} + mk\Omega^{m-1}\frac{\partial \Omega}{\partial x} = 0 \Rightarrow \frac{\partial \Omega}{\partial t} \cdot m\frac{Q}{\Omega}\frac{\partial \Omega}{\partial x} = 0$$

$$\Rightarrow \frac{\partial \Omega}{\partial t} + mV\frac{\partial \Omega}{\partial x} = 0$$

Con $mV = c(\Omega)$ celerità:

$$\frac{\partial \Omega}{\partial t} + c\frac{\partial \Omega}{\partial x} = 0$$

cambio variabile

$$\begin{cases} \xi = x - c_0 t \\ \tau = t \end{cases}$$

$$\Rightarrow \frac{\partial \Omega}{\partial t} = \frac{\partial \Omega}{\partial \xi}\cdot\underbrace{\frac{\partial \xi}{\partial t}}_{=-c_0} + \frac{\partial \Omega}{\partial \tau}\cdot\underbrace{\frac{\partial \tau}{\partial t}}_{=1}$$

$$\Rightarrow \frac{\partial \Omega}{\partial x} = \frac{\partial \Omega}{\partial \xi}\cdot\underbrace{\frac{\partial \xi}{\partial x}}_{=1} + \frac{\partial \Omega}{\partial \tau}\cdot\underbrace{\frac{\partial \tau}{\partial x}}_{=0}$$

Quindi:

$$-\frac{\partial \Omega}{\partial \xi}c_0 + \frac{\partial \Omega}{\partial \tau} + c_0\frac{\partial \Omega}{\partial \xi} = 0 \Rightarrow \frac{\partial \Omega}{\partial \tau} = 0$$

Ω dipende solo da ξ non da $\tau \Rightarrow x_1 - x_0 = c_0(t_1 - t_0)$

Per risolvere l'equazione $\frac{\partial \Omega}{\partial t} + c(\Omega)\cdot\frac{\partial \Omega}{\partial x} = 0$ introduciamo un'equazione caratteristica $c(\Omega) = \frac{dx}{dt}$:

$$\frac{\partial \Omega}{\partial t} + \frac{\partial \Omega}{\partial x}\cdot\frac{dx}{dt} = \frac{\partial \Omega}{\partial t}\cdot dt + \frac{\partial \Omega}{\partial x}\cdot dx = 0$$

$$\Rightarrow d\Omega = 0 \Leftrightarrow \Omega = \cos t$$

L'area della sezione Ω si mantiene costante.